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Eikonal Approach to Planck Scale Physics

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Abstract

We consider gravitational scattering of point particles with Planckian centre-of-mass energy and fixed low momentum transfers in the framework of general relativity and dilaton gravity. The geometry around the particles are modelled by arbitrary black hole metrics of general relativity to calculate the scattering amplitudes. However, for dilaton gravity, this modelling can be done *only* by extremal black hole metrics. This is consistent with the conjecture that extremal black holes are elementary particles.

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I. INTRODUCTION

It is well known that the quantum effects of gravity come into play at the Planck energy scale (which is about $M_{pl} \sim 10^{19}\ GeV$). The space-time curvatures become too large for classical general relativity to hold good at this scale. There has been various attempts to understand these effects and to formulate a quantum theory of gravitation. Notable among them are string theory and the Ashtekar formalism. However, till date, there is no fully satisfactory renormalisable theory of quantum gravity. We address the issue of Planck scale physics in the context of particle scattering via gravitational interaction at very high energies. The kinematics of particle scattering can be expressed by the two independent Mandelstam variables s, and t, which are Lorentz scalars. They are respectively the squares of the centre-of-mass energy and the momentum transfer in the scattering process. Newton's constant G being a dimensional constant and equal to M_{pl}^{-2}, the Planck scale can arise in two ways, either when $Gs \sim 1$ or when $Gt \sim 1$. Thus, the most general quantum gravitational scenario involves both s and t approaching the Planck scale.

The eikonal approximation, on the other hand, is characterised by scattering at high s and fixed low t. Physically, this signifies scattering of particles at very high velocities (and kinetic energies) and at large impact parameters, such that the interaction is weak and the particles deviate slightly form their initial trajectories. In other words, they scatter almost in the forward direction. We would restrict our analyses to this approximation and try to extract whatever information is available about the Planck scale effects as reflected in the scattering amplitudes. The motivation to study this kinematical regime is, as we shall see, that the scattering amplitudes in this approximation can be exactly calculated and expressed in a closed form. These of course become significant only at Planckian centre-of-mass energies and we can obtain some quantitative results about quantum gravity in this kinematical domain.

Without loss of generality, two-particle scattering processes are considered. An inertial frame is chosen, in which one of the particles move at almost the speed of light and

the other is relatively slow. Then one of these point particles is modelled as the source of an appropriate metric of general relativity. For example, neutral particles are modelled by Schwarzschild metric and electrically or magnetically charged particles are modelled by Reissner-Nordström metric. Finally, the quantum mechanical wave-function of the other particle in the background of this space-time is analysed to deduce the corresponding scattering amplitude. The effect of electromagnetism is also studied when the particles also carry electric and/or magnetic charges.

Next, we replace these black hole metrics by their counterparts in dilaton gravity, i.e. those that arise in low energy string theory. Here, we see that the above mentioned modelling cannot be done by generic black holes as was the case for general relativity. If we consider charged particles, for example, the modelling can be successfully done only by extremal black holes to be able to calculate the scattering amplitudes. This supports the conjecture that extremal black holes can indeed be *identified* with elementary particles.

II. EIKONAL SCATTERING IN GENERAL RELATIVITY

We begin by considering the scattering of neutral point particles. The space time around any such particle, when it is static, is obtained by solving Einstein's equations and given by the well known Schwarzschild metric:

$$ds^2 \;=\; \left(1-\frac{2GM}{r}\right)dt^2-\left(1-\frac{2GM}{r}\right)^{-1}dr^2-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right) \;. \qquad (1)$$

To obtain the corresponding space-time when the particle is moving at a very high velocity, say along the positive z-axis, we perform a Lorentz transformation on the metric tensor components with the velocity parameter β according to:

$$t' \;=\; \gamma\left(t-\beta z\right) \;,$$

$$z' \;=\; \gamma\left(z-\beta t\right) \;,$$

where $\gamma = 1/\sqrt{1-\beta^2}$. Simultaneously, the mass M is parametrised as

$$M \;=\; \frac{P_0}{\gamma} \;.$$

This is to ensure that the energy of the boosted particle remains finite and equals P_0. The metric tensor components transform as a symmetric second rank tensor. Dropping the primes and taking the limit $\beta \to 1$, the final form of the metric is:

$$ds^2 \;=\; dx^- \left[dx^+ - \frac{2GP_0}{|x^-|} dx^- \right] - dx_\perp^2 \;\;, \tag{2}$$

where $x^\pm \equiv t \pm z$, the light cone coordinates. The above geometry was first obtained in [1] and then in [2]. Defining the new coordinate differentials $d\tilde{x}^\mu$ as

$$d\tilde{x}^+ \;=\; dx^+ - \frac{2GP_0}{|x^-|} dx^- \;, d\tilde{x}^- \;=\; dx^- \;, d\tilde{x}_\perp \;=\; dx_\perp \;,$$

Eq.(2) can be re-written as,

$$ds^2 \;=\; d\tilde{x}^- d\tilde{x}^+ - d\tilde{x}_\perp^2 \;. \tag{3}$$

The above form of the infinitesimal line element seems to indicate that we have simply arrived at flat space-time by a coordinate re-definition. However, writing the finite forms of these re-definitions, we get:

$$\tilde{x}^+ \;=\; x^+ - 2GP_0 \theta \left(x^- \right) \ln x_\perp \;, \tag{4}$$

$$\tilde{x}^- \;=\; x^- \;, \tag{5}$$

$$\tilde{x}_\perp \;=\; x_\perp \;. \tag{6}$$

Note that the transformations are continuous everywhere except at $x^- = 0$, which is the trajectory of the boosted particle, where there is a step function discontinuity in the coordinate x^+. Calculation of the Riemann-Christoffel curvature tensor reveals that they are Dirac-delta functions (derivatives of the θ functions), which are non-vanishing only at $x^- = 0$. Thus, all space-time curvatures are localised on the two-dimensional transverse plane, perpendicular to the trajectory of the boosted particle and travelling along with it. We call this infinite plane the *shock-wave*. The space-time in front of and behind the shock-wave is Minkowskian. It is analogous to the case of a boosted charged particle, where the electromagnetic fields tend to become concentrated along the direction perpendicular to

the particle trajectory, and in the limit $\beta \to 1$, they are completely localised on the plane fronted (electromagnetic) shock-wave. Note that the coordinate x^- is however, continuous at all points and serves as a bonafide affine parameter for any test particle in the above background. The classical geometry is depicted in Fig(1), which can be thought of as two Minkowski spaces, pertaining to $x^- < 0$ and $x^- > 0$ respectively and glued along the null plane $x^- = 0$, which is the trajectory of the boosted particle and the shock-wave.

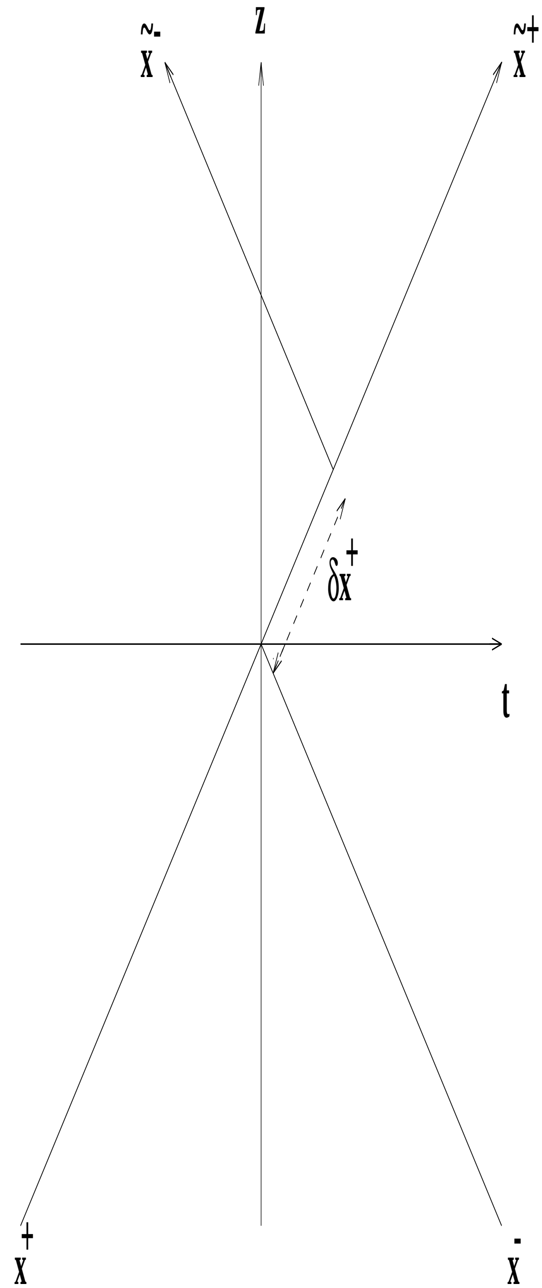

Fig.1: Shock-wave geometry of boosted Schwarzschild metric.

Having found the geometry of the luminal particle, now we concentrate on the other particle, assumed to be relatively slow. It serves as the test-particle in the above background. Before the shock wave comes and hits this particle, it is free from any interactions, and its

wave-function is given by

$$\psi_< \;=\; e^{ip\cdot x} \;=\; e^{i\left[p_+x^+ + p_-x^- + \vec{p}_\perp\cdot\vec{x}_\perp\right]} \quad , \qquad x^- < 0 \quad . \tag{7}$$

The moment when it is hit by the shock-wave, the x^+ coordinate undergoes a discrete shift given by Eq.(4) and the wave function picks up a space-time dependent phase factor. Simplifying, we get the final wave function to be:

$$\psi_> \;=\; e^{-iGs\ln x_\perp^2}\;\psi_< \; . \tag{8}$$

Here we have used the identity $2p_-p_0 = s$. To calculate the scattering amplitude from this wave function, we expand it in terms of a complete set of momentum eigenstates and perform an inverse Fourier transform to obtain the expansion coefficients. The latter can be identified with the scattering amplitude modulo kinematical factors. The detailed calculation is done in ref. [3], and the final result is:

$$f(s,t) = \frac{Gs}{t}\frac{\Gamma(1-iGs)}{\Gamma(1+iGs)}\left(\frac{-1}{t}\right)^{-iGs} \quad . \tag{9}$$

Note that the above expression is simply the Rutherford Scattering amplitude with the gravitational coupling constant $-Gs$ replacing its electromagnetic counterpart α. As advertised, it captures the gravitational interactions between point particles at the Planck scale and is insignificant for sub-Planckian energies, when $Gs \ll 1$.

Without going into the details, which the reader will find in [5], we summarise in brief the situation when electromagnetic interactions are included in the scattering process. That is, the scattering particles also carry electric or magnetic charges. If they carry electric charges e and e', then the scattering amplitude is modified to:

$$f(s,t) = \frac{Gs-ee'}{t}\frac{\Gamma\left(1-i(Gs-ee')\right)}{\Gamma\left(1+i(Gs-ee')\right)}\left(\frac{-1}{t}\right)^{-i(Gs-ee')} \quad . \tag{10}$$

The remarkable fact about this expression is that it can be obtained from the purely gravitational result in Eq.(9) simply by making the replacement $Gs \to Gs - ee'$. This means that the gravitational and electromagnetic coupling constants simply add up to give the effective

coupling and there is no interference between them. This is quite unique and holds only in the eikonal approximation, because as we know, the two forces do affect each other in a non-trivial manner in generic cases. This decoupling is reminiscent of the Newtonian limit, where gravitation and electromagnetic interactions can be assumed not to affect each other. However, as far as the velocities and the energies of the particles are concerned, we are far removed from the Newtonian (low velocity) regime.

If, on the other hand, one of the particles carry an electric charge e, and the other a magnetic charge g, then the scattering amplitude is [6]:

$$f(s,t) \; = \; \left(\frac{n}{2} - iGs\right) \frac{\Gamma\left(\frac{n}{2} - iGs\right)}{\Gamma\left(\frac{n}{2} + iGs\right)} \left(\frac{-1}{t}\right)^{1-iGs} \tag{11}$$

Here, the two couplings do not add up in a simple manner as in the previous case, but the same in not expected intuitively, because with magnetic monopoles, the interaction is no longer central in nature like gravitation or electromagnetism involving charges only. Comparing Eqs. (10) and (11), we find that the electromagnetic contribution to the former is insignificant (with ee' typically of the order of the fine structure constant $1/137$), while for the latter, it is comparable to the gravitational contribution (both couplings being of the order of unity). In short, gravitation dominates overwhelmingly over electromagnetism at Planckian energies in the absence of magnetic monopoles. Introduction of the latter entails drastic changes in the result.

III. SHOCK WAVES IN DILATON GRAVITY

As noted earlier, the results in the previous section were derived in the framework of general relativity, and all the black hole metrics used to model the particles satisfy the Einstein's equations. Now, string theory, in the low energy limit provides us an alternative theory of gravitation, known as *dilaton gravity*, where in addition to the metric tensor, a scalar field called the **dilaton** is also an independent degree of freedom. We will not go into the details as to how this theory emerges as a low energy *effective* theory from string

theory. Instead, we will analyse the scattering situation envisaged in section II using dilaton gravity. We would like to investigate whether the scattering amplitudes are modified in this framework, and whether the electromagnetic and gravitational decoupling still hold good. We will see that modelling the scattering particles by dilatonic black holes poses some generic pathologies, which are removable only under certain specific conditions and when these are satisfied the decoupling exists just as in the case of general relativity. The counterpart of the Reissner-Nordström metric in dilaton gravity is given by the following expression (in the so-called 'string metric') [7]:

$$ds^2 \;=\; \left(1 - \frac{\alpha}{Mr}\right)^{-1} \left[\left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2\right] - r^2 d\Omega^2 \;, \tag{12}$$

where $\alpha = Q^2 e^{2\phi_0}$, Q being the electric charge and ϕ_0 the asymptotic value of the dilaton field. Note that setting $Q = 0$ reproduces the Schwarzschild metric, which means that the dilaton field has non-trivial effects only when we consider charged solutions. Like the Reissner-Nordström solution, the above metric has two horizons r_+ and r_-, given by:

$$r_+ \;=\; 2GM \;,$$

$$r_- \;=\; \frac{\alpha}{M} \;.$$

However, here a crucial difference is that the inner horizon r_- is a space-time singularity, where the curvature tensor blows up. For black holes of large masses, this singularity is hidden behind the event horizon r_+ and there is no naked singularity, which however develops as the mass decreases. We will see that this singularity plays a crucial role in eikonal scattering. Analogous to the Schwarzschild or the Reissner-Nordström metric, we apply a Lorentz boost to the dilaton gravity metric along the positive z-axis and take the limit $\beta \to 1$. The resultant metric is of the form [7]:

$$ds^2 \;=\; dx^- \left[\frac{1 - \alpha/2P_0|x^-|}{1 - \alpha/P_0|x^-|}\right] \left[dx^+ - \frac{4GP_0/|x^-|}{1 - \alpha/P_0|x^-|}\; dx^-\right] - dx_\perp^2 \;. \tag{13}$$

As before, to express this in a form representing Minkowski space, we define the new coordinates

$$d\tilde{x}^- = dx^- \left[\frac{1 - \alpha/2P_0|x^-|}{1 - \alpha/P_0|x^-|} \right] \tag{14}$$

$$d\tilde{x}^+ = dx^+ - \frac{4GP_0/|x^-|}{1 - \alpha/P_0|x^-|}\, dx^- \quad . \tag{15}$$

Note that here both the coordinates x^- and x^+ are discontinuous for non-vanishing α. This is rather disturbing, because as we saw in the previous section, x^- served as the continuous affine parameter for the test particle, and a discontinuity in it signals a breakdown in the description of the evolution of the particle in terms of geodesics. This is indeed confirmed by writing the classical geodesic equations of the test particle in the background of the boosted dilaton metric and trying to solve it perturbatively in powers of the mass M. The failure of the latter indicates that the null geodesics are incomplete in this case and the curvature singularity at $r^- = \alpha/M$ shows up as an extended naked singularity in the boosted limit and renders eikonal scattering impossible. The classical geometry in this case is shown in Fig.(2), where the rectangular shaded region denotes the finite region of singularity that originated from the singular horizon r_-. Thus, we see that modelling the particles by dilaton gravity metric gives rise to singularities which makes the subsequent calculation of scattering amplitudes impossible.

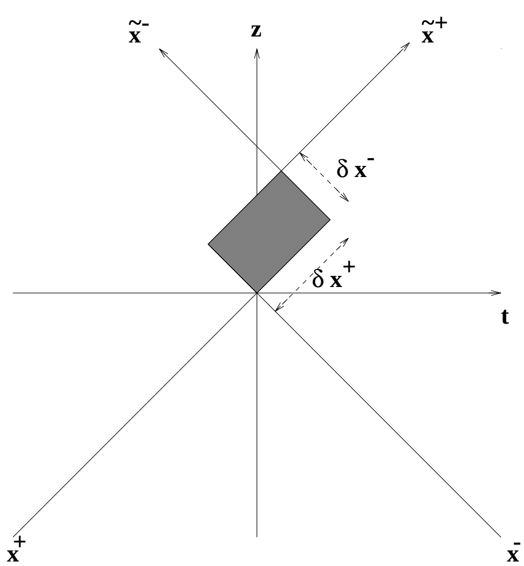

Fig.1: Shock-wave geometry of boosted non-extremal dilaton gravity metric.

To circumvent this difficulty, we can try various possibilities. In particular, we can examine the case by imposing the *extremal limit* $r_+ = r_-$. Then, the metric assumes the form

$$ds^2 \;=\; dt^2 - \frac{dr^2}{\left(1 - \frac{2GM}{r}\right)^2} - r^2 d\Omega^2 \;, \tag{16}$$

which is perfectly regular everywhere. The curvature singularity has simply disappeared when the two horizons coincide ! This is precisely the motivation behind considering this limit. Note that the metric above is entirely distinct from the Schwarzschild metric. However, on performing the usual Lorentz boost on it along with the limit $\beta \to 1$, it can be easily verified that the boosted version coincides with the metric (2). This is a pleasant surprise, because now the test particle will 'see' an identical background as in the case of Schwarzschild, and the corresponding scattering amplitude is simply the eikonal result (9).

Thus it is clear that the curvature singularity shows up as an extended singular region and makes it impossible to calculate scattering amplitude. The extremal limit, on the other hand, plays a special role whose imposition reproduces the elegant eikonal results. In the next section, we will probe into the details of the role of this singularity by invoking a different formalism.

IV. SCATTERING IN STATIC DILATON GRAVITY BACKGROUND

In this section, we try to establish rigorously, the role of the naked singularity in dilaton gravity metric in eikonal scattering. Here we consider the scattering of the fast particle in the background of the other static particle. Note that the definitions of 'source' and 'test' has been reversed. The Klein-Gordon equation for the wave function ϕ of the fast particle is given by:

$$g_{\mu\nu} \; D^\mu D^\nu \Phi \;=\; 0 \;, \tag{17}$$

where D^μ denotes the covariant derivative. The above equation can be simplified to

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi \right) \;=\; 0 \;. \tag{18}$$

We assume a solution of the form

$$\Phi \;=\; f(r)\; Y_{lm}(\theta,\phi)\; e^{iEt}\;, \tag{19}$$

where we have split the wave function into a radial part, the spherical harmonics and the exponential of asymptotically measured energy E. This decomposition results from the spherically symmetric and static nature of the dilaton gravity metric (12). With this ansatz, the following radial equation is obtained from Eq.(18) [9]:

$$r^2\Lambda\frac{d^2f}{dr^2}+\frac{d\,(r^2\Lambda)}{dr}\,\frac{df}{dr}-\left[\frac{l(l+1)}{\Delta}-\frac{E^2r^2}{\Lambda}\right]\;f\;=\;0\;, \tag{20}$$

where we have defined the quantities $\Lambda \equiv 1-2GM/r$ and $\Delta \equiv 1-\alpha/Mr$. As expected, we recover the radial equation for Schwarzschild background for $\alpha=0, \Delta=1$, and the subsequent scattering amplitude (9) [10]. However, for generic values of Δ, it can be shown that the above radial equation does not admit of a solution at all. This follows from an elementary theorem in ordinary differential equations since the coefficient of f in eq.(20) suffers an infinite discontinuity at $\Delta = 0$. Thus, as in the previous section, we conclude that the dilaton gravity metric (12) cannot be used to successfully model the high energy particles. Moreover, we are now in a position to understand the physical origin of the pathology. The curvature singularity at $r_- = \alpha/M$ grows indefinitely large as we take the limit $M \to 0$ and eventually fills all space around the dilatonic particle. Thus, the other particle, at arbitrary impact parameter, is forced to hit this singularity and get trapped, signaling the breakdown of the scattering process. This was reflected in the non-existence of solutions of the classical geodesic equations in section III. Imposing the extremal limit, on the other hand, eliminated this singularity, and thus the scattering amplitude became calculable, which yielded the eikonal result. This can be verified using the Klein-Gordon radial equation also. In the extremal limit, Eq.(20) assumes the form [10]:

$$\frac{d^2f}{dr^2}+\frac{1}{r^2\Lambda}\frac{d\,(r^2\lambda)}{dr}\frac{df}{dr}-\frac{1}{\Lambda^2}\left[\frac{l(l+1)}{r^2}-E^2\right]\;f\;=\;0\;. \tag{21}$$

The above radial equation can be expanded in powers of GM/r and this recovers the Schwarzschild radial equation to the lowest order. The scattering amplitude (9) follows

immediately. This re-emphasises the importance of the extremal limit for Planckian scattering via dilaton gravity.

V. PERTURBATIVE APPROACH

In the previous two sections, we have explicitly used a classical solution of the low energy string effective action to arrive at the scattering amplitudes (in the extremal limit). However, the problem can be approached without the help of such explicit solutions, at the level of the action itself. Historically, the eikonal scattering amplitude was derived in the context of quantum electrodynamics by summing an infinite subset of Feynman diagrams, known as *ladder diagrams*, with certain kinematical restrictions on the matter propagators. It was shown that this infinite sum can be expressed in a neat closed form [12]. A crucial assumption required to arrive at the eikonal result is the assumption that the scattering particles have well defined *classical* trajectories, which differ slightly from the free particle trajectories. The sum of ladder diagrams were seen to converge for gravitational scattering as well in [13] which reproduced the amplitude (9) exactly. For dilaton gravity, however, the assumption regarding classical trajectories is invalidated because, as we saw in the previous sections, an incoming particle is swallowed up by the expanding curvature singularity and there are no well defined scattering solutions. Thus, a priori, it seems impossible to construct and calculate ladder diagrams from dilaton gravity action given by:

$$S = \int d^4x \sqrt{-g} e^{-2\phi} \left[-\frac{R}{G} + F_{\mu\nu}F^{\mu\nu} + 2\partial_\mu\phi\partial^\mu\phi \right] . \tag{22}$$

First, we simplify the action by linearising the metric as well as the dilaton field,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} , \tag{23}$$

$$\phi = \phi_0 + f , \tag{24}$$

where $\eta_{\mu\nu}$ is the flat Minkowskian metric and ϕ_0 is some constant. Retaining terms to leading order in these quantum fluctuations, the action (22) reduces to:

$$
\begin{aligned}
S \;=\; & \frac{e^{-2\phi_0}}{G}\int d^4x(1-2f)\frac{1}{8}h_{\mu\nu}\left[\eta^{\mu\lambda}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\lambda}-\eta^{\mu\nu}\eta^{\lambda\sigma}\right]\Box h_{\lambda\sigma} \\
& - e^{-2\phi_0}\int d^4x\left(1+\frac{1}{2}h_\alpha{}^\alpha\right)(1-2f)\left[-4\partial_\mu f\partial^\mu f+F^2+\partial_\mu\chi\partial^\mu\chi\right] \;. \qquad (25)
\end{aligned}
$$

Here we have also included the action for the massless matter field χ representing the scattering particles. In addition to the graviton and matter propagators and the matter-graviton interaction vertex already calculated in [13], now we have a dilaton propagator and a matter-dilaton interaction vertex. The factors associated with them can be read of fom the linearised action, and turns out to be $-i/(p^2+m^2-i\epsilon)$ and $-2p\cdot p'$ respectively, where p and p' are the momenta associated with the external matter lines. With these, the new infinite set of ladder diagrams with dilaton exchanges can be computed in a straightforward manner. The details of the calculation is done in [9], and the final result is:

$$
i\mathcal{M} \;=\; \frac{ip_1^2p_2^2}{-t}\frac{\Gamma(1-ip_1^2p_2^2/Ep)}{\Gamma(1+ip_1^2p_2^2/Ep)}\left(\frac{4}{-t}\right)^{-i\frac{p_1^2p_2^2}{Ep}} \;. \qquad (26)
$$

Now, if we make the momenta p_1 and p_2 on-shell and replace them by m^2 and eventually take the massless limit $m\to 0$, then we see that the dilaton amplitude vanishes identically and we are simply left with the gravitational result (9)!

At this point, we investigate the circumstances under which the action can be linearised, because without the latter, the eikonal sum can never be attempted. The metric $g_{\mu\nu}$ is linearised under the assumption that there are small graviton fluctuations over a Minkowskian background. As for the dilaton field, we can try to estimate its quantum fluctuation f by looking at the classical solution for ϕ obtained by minimising the action (22):

$$
e^{2\phi} \;=\; e^{2\phi_0}\left(1-\frac{\alpha}{Mr}\right) \;, \qquad (27)
$$

which implies that the fluctuations over ϕ_0 are of the order of

$$
f\sim|\phi-\phi_0|\sim|\ln(-\alpha/Mr)| \;. \qquad (28)
$$

Smallness of this requires $|\alpha/Mr|\to 0$, or in other words *alpha* should scale at least as M^2 as we take $M\to 0$. But this is equivalent to the extremal limit, when $\alpha=2GM^2$! Thus,

even in a solution-independent approach, where we look at the action and impose certain restrictions on it, the extremal limit seems to emerge in a natural way, if we want to obtain well defined scattering amplitudes.

VI. CONCLUSION

Our analysis of Planckian scattering in the light of dilaton gravity unambiguously point to the fact that there are important constraints to be satisfied while trying to model the scattering particles by a suitable metric. Namely, the extremality condition should be necessarily imposed on the parameters to obtain physically meaningful scattering amplitudes. Otherwise, the curvature singularity at a finite radius inevitably shows up as pathologies in the calculations. In the shock-wave approach, where we boosted the dilaton gravity metric, this was seen to give rise to the a discontinuous affine parameter. Next, while trying to solve the Klein-Gordon equation in the background of the above metric, we saw that there were no scattering solutions to the corresponding radial equation. Finally, in the approach of perturbation theory, we showed that the eikonal scattering amplitude can be reproduced if and only if the dilaton field was linearised and the lowest order terms in its quantum fluctuation is retained in the action. Once again, this linearisation is consistent with the extremality condition. There is yet another solution-independent way to arrive at identical conclusions, starting from the dilaton gravity action, where one uses the so called 'Verlinde-scaling' to incorporate the eikonal kinematics. The interested reader may refer to [9] for a detailed discussion of this approach. The important point to note is that no such restrictions were ever necessary in the general relativistic framework to calculate scattering amplitudes. Thus, it is perhaps correct to say that the theory of gravity that emerges from string theory incorporates certain problematic features, at least in the context of Planckian scattering. But the same theory contains the cure to this problem also, namely in the form of the extremal limit! The latter constraint, once imposed, removes the pathologies altogether and reproduces the finite amplitudes of general relativity. It is also curious to note the consis-

tency of these results with the well known conjecture that extremal black holes are actually elementary particles [15]. Here, we are considering scattering of point particles, which can also be regarded as ‘elementary’. Thus, it seems logical in the spirit of the conjecture, to model them as extremal black holes.

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